## A NEURONAL TOOL FOR AVIRIS HYPERSPECTRAL UNMIXING

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### 1.INTRODUCTION.

Nowadays, in most of hyperspectral signal processing applications it is necessary to determine and quantify the components in a composite pixel spectrum obtained from a given mixture of elements. This problem is known as *Hyperspectral Unmixing*.

Formally, the problem may be considered as follows: Assuming we know the spectra of *K* elements (*endmembers*), we must determine the unknown composition of a *cocktail* of the mentioned elements using the radiation spectrum of a pixel with this mixture.

The conventional digital algorithms to solve this problem is fairly slow, since serial computation is implied. The difficulty increases in the presence of Miscalibration Problems on the Hyperspectral Sensor (MPHS).

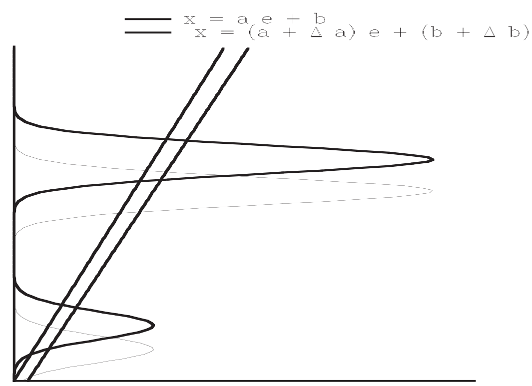


**Figure** Error! Unknown switch argument.. *MPHS*

The main intention of this paper is to explore the possibility of using a Neural Networks Methodology to obtain a reliable, robust and efficient solution to the *Hyperspectral unmixing with MPHS*, based on the inherent parallelism of neural networks.

A method based on the Optical Neural Network to solve the hyperspectral unmixing was presented by Barnard and Cassasent (Barnard et al., 1989). One of the main inconveniences of this approach is the lack of uniqueness of the proposed solution, and another is the MPHS no assumption.

The possibility of using the Multiple Regression Theory to solve the same problem, granting an optimal solution in terms of uniqueness, has been developed by Díaz et al. (Díaz et al., 1992) This approach is based on the use of the Pseudo-Inverse Matrix, supported by a Linear Associative Memory, built using Pyle's algorithm. One of the advantages of this method is the MPHS treatment.

## 2. ALGORITHMIC METHOD.

In order to describe the algorithmic method suggested in the present work, it must be taken into account that a Composite Spectrum $\boldsymbol{x}$ may be seen as a *N-dimensional* vector, which is built sorting the emission intensities associated to each energy channel vs. the channel number, where $N$ is the total number of energy channels:

$$x = \left[x_0, x_1, \cdots, x_{N-1}\right]^T$$
$$x_n \; ? \; 0 \qquad 0 \le n \le N-1$$

(Error! Unknown switch argument.)

being $\boldsymbol{x_n}$ the intensity measured as the number of photons whose energy is comprised in the *n-th* energy channel's interval.

In this way, a *Endmember Spectrum* is a spectrum of the same nature, but produced by an *Individual Source*. We denote these spectra as $\boldsymbol{r_k}$, with $0 \le k \le K-1$. The set of $K$ endmembers vectors is named the *Reference Set*, and it must be evaluated in advance. For the sake of compactness, it is denoted as a *Reference Matrix* $\boldsymbol{R}$ composed by the reference column vectors:

$$R = \left[r_0, r_1, \cdots, r_{K-1}\right]$$

(Error! Unknown switch argument.)

In a general sense, the set of Composite Spectra is the set of all possible spectra that may be produced by a linear combination of all elements belonging to the *Reference Set*. When the *Reference Set* is composed by $K$ linearly independent vectors, this would result in a *K-dimensional Vector Space*, integrated by all the vectors $\boldsymbol{y}$ given by:

$$y = Rc = \sum_{i=0}^{K-1} c_i r_i$$

(Error! Unknown switch argument.)

where $\boldsymbol{c}$ is the *Contribution Vector*, defined as:

$$c = \left[c_0, c_1, \cdots, c_{K-1}\right]^T$$
$$c_k \; ? \; 0 \qquad 0 \le k \le K-1$$

(Error! Unknown switch argument.)

and where every contribution $\boldsymbol{c_i}$ is a function of the relative intensities of the i-Endmember on the Composite Spectra. Our goal is to estimate $\boldsymbol{c}$ assuming that $\boldsymbol{R}$ and $\boldsymbol{y}$ are known.

The general solution method proposed here is based on the Hopfield Recurrent Neural Network (HRNN). It is a flexible, efficient and robust approach to solve the problem (Pérez et al., 1995). The Gradient Method for minimizing errors is used to assure the convergence of the algorithm. The use of this model is fully justified when the spectrum formation in the *Hyperspectral unmixing* is a linear process (Pérez et. al.,1996)(Aguilar et al., 1998)

## 3. HRNN FOR MPHS SPECTRA.

In order to explore the performance of the HRNN algorithm to solve the hyperspectral unmixing with MPHS, we must know that when MPHS are implied in the hyperspectral unmixing, due to numerous causes, the Instrumentation Transfer Function (ITF) that generates the different spectra, even though the spectrum is composed of an only reference, it can not be considered constant along the time, assuming different values of ITF for each measured spectra.

As far as the present research is concerned, we assume that the main characteristics of the ITF, the gain $\boldsymbol{a}$ and the offset $\boldsymbol{b}$, will only appear in the formation of the composite spectrum.

When drifts in gain and offset are considered, the obtained spectrum $\boldsymbol{y(n)}$ will be related with the zero-drift one $\boldsymbol{y(m)}$ as follows:

$$y(n) = y(am + b)$$

(Error! Unknown switch argument.)

where $\boldsymbol{m}$ is the bin number associated with the energy channel interval E-δE to E+δE.

Initially, we assume, without a loss of generality, that both $\boldsymbol{n}$ and $\boldsymbol{m}$ are real continuous variables. Typically, under the reference conditions we take the gain $\boldsymbol{a}$=1 and the offset $\boldsymbol{b}$=0. Otherwise we try to approximate the spectrum taken under different conditions using the first order Taylor's Expansion of the equation **(5)**, which may be written as:

$$y(n) \cup y(m) + (a-1)my'(m) + by'(m)$$
$$where \quad y'(m) = \frac{dy}{dm} \qquad \textbf{(}\text{Error! Unknown switch argument.}\textbf{)}$$

This expression approximates the spectrum taken under non-zero drifts ***y(n)*** as a linear expansion of the same spectrum taken under ideal zero drifts ***y(m)***, its first derivate multiplied by the channel number, and its first derivate.

Using the expressions **(3)** and **(4)**, we can express the ***y(n)*** spectrum as:

$$y(n) = \mathop{\square}_{i=0}^{K-1} c_i r_i + (a-1) \mathop{\square}_{i=0}^{K-1} c_i r_i^{'}(m) =$$
$$= \square \left( c_i^0 r_i^0 + c_i^1 r_i^1 + c_i^2 r_i^2 \right) \qquad \textbf{(}\text{Error! Unknown switch argument.}\textbf{)}$$
$$where \quad c_i^0 = c_i \quad c_i^1 = (a-1)c_i \quad c_i^2 = bc_i$$
$$and \quad r_i^0 = r_i \quad r_i^1 = mr_i^{'}(m) \quad r_i^2 = r_i^{'}(m)$$

Then, we consider the generalized contributions vector as:

$$c^* = \left( c_0^0 c_0^1 c_0^2, \cdots, c_i^0 c_i^1 c_i^2, \cdots, c_{k-1}^0 c_{k-1}^1 c_{k-1}^2 \right) \qquad \textbf{(}\text{Error! Unknown switch argument.}\textbf{)}$$

In order to solve the MPHS problem using the Taylor's expansion **(6)** and taking into account **(7)**, it is necessary to expand the Reference Set from order K to order 3K to include the first derivate $\boldsymbol{r_0^1}$ and $\boldsymbol{r_0^2}$.

The spectrum drifts can be inferred from the set of generalised contributions $\boldsymbol{c^*}$. This possibility can be exploited to correct the non-zero drift composite spectrum ***y*** partially, which may be recursively processed with the HRNN to obtain a new and more refined estimation of the contributions of the components and drifts ***a*** and ***b***.

To sum up, the recursive application of the HRNN using the *Expanded Reference Set* allows us to obtain both quantified and approximations of the influence of each component and an estimation of the drifts values ***a*** and ***b***.

## 4. RESULTS.

Through the present work, we have evaluated the performance of the HRNN algorithm with non-zero drift spectra, expanding the Reference Set. The obtained results show that applying recursively the HRNN algorithm to problem spectrum, we obtain gain and offset values that allow us to rebuild a new spectrum $\boldsymbol{y}^{\wedge}$ being an optimal approach to ***y.***

To evaluate the HRNN performance we use the **sin** of the angle between ***y*** and ***y^***.

Some experiments have been designed to measure the influence of different parameters:

- *Level of Noise in the Mixture Spectra.*
- *Proportion of Elements in the Mixture*
- *Correlation between Components.*

**Level of Noise in the Mixture Spectra.**

The figure **2** represents the first set of experiments. It measures the effects of noise in the behaviour of the method.

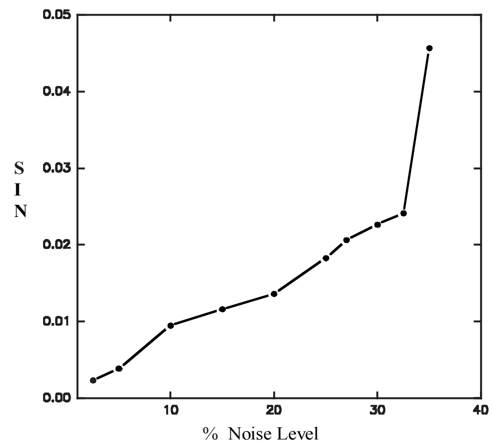


**Figure. 2** *Level of Noise in the Mixture Spectra.*

**Proportion of Elements in the Mixture.**

In the figure**.**3 we can see that the proportion between components has a little influence in the resulting error.

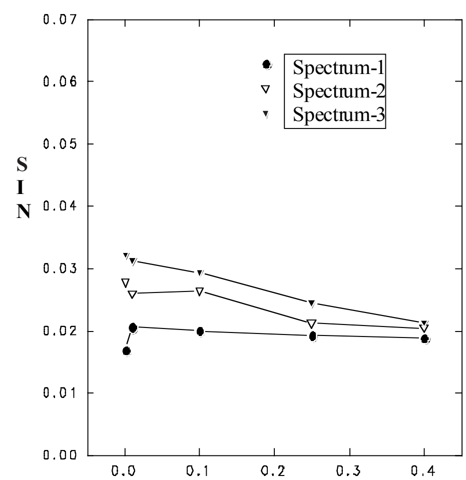


**Figure. 3.** *Proportion of Elements in the Mixture.*

**Correlation between Components.**

The figure 4 shows the ability of the method to distinguish between two different spectra as a function of their relative correlation coefficient

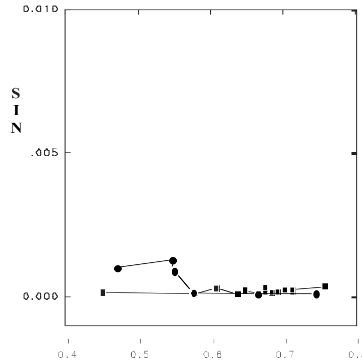


**Figure. 4.** *Correlation between Components.*

### 5. SUMMARY AND CONCLUSIONS.

We have developed a neural tool to apply the HRNN method to multispectral images; this method seems more reliable than other traditional methods.

The method is able to resolve AVIRIS images at a reasonable computational cost, obtaining images of the proportion of each endmember on the original image.

Figure 5 Original Band of the AVIRIS image

Figure 6 Grey scaled Endmember contribution

Figure 5 shows the AVIRIS band image where the HRNN neuronal tool was applied.

Figure 6 shows the resulting image of the quantification of one endmember of this image; higher pixel values are associated with high contribution zones of the component.

### REFERENCES:


Barnard E. and Cassasent P., August 1.989, 'Optical Neural Net for Classifying Imaging Spectrometer'. *Applied Optics,* vol. 18, n. 15, pp. 3129-3133.

Díaz J.C., Aguayo P., Gómez P., Rodellar V. and Olmos P., August 1.992, 'An Associative Memory to Solve the Mixture Problem in Composite Spectra'. *Proc. of the 35th Midwest Symposium on Circuitsand System* Washington DC, pp. 422-428.

Pérez R.M., Martínez P., Silva A. and Aguilar P.L. , May 1.996, 'Influence of the Fixed Point Format on the Accuracy of the Neural Network Solution to the Mixture Problem'. *Proc. of the 3rd Advanced Training Course Mixed Design of Integrated Circuits and Systems*, Lodz (Poland), May 1996.

Pérez R.M., Martínez P., Martínez L., Díaz J.C., Rodellar V. and Gómez P., April 1.995, 'A Hopfield Neural Network to Solve the Mixture Problem'. *Proc. of the VI Spanish Symposium on Pattern Recognition and Image Analysis*, Córdoba, pp. 744-745.

Aguilar P.L., Pérez R.M., Martínez P., Bachiller P. and Merchán A., February 1.998, 'Spectra evaluation and recognition in the mixture problem using SOFM algorithm'. *Proc. of the EIS 98, Vol 2,*Tenerife, 118-124